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Design of Shear Deformable Antisymmetric Angle-Ply Laminates to Maximize the Fundamental Frequency and Frequency Separation

National Research Inst. for
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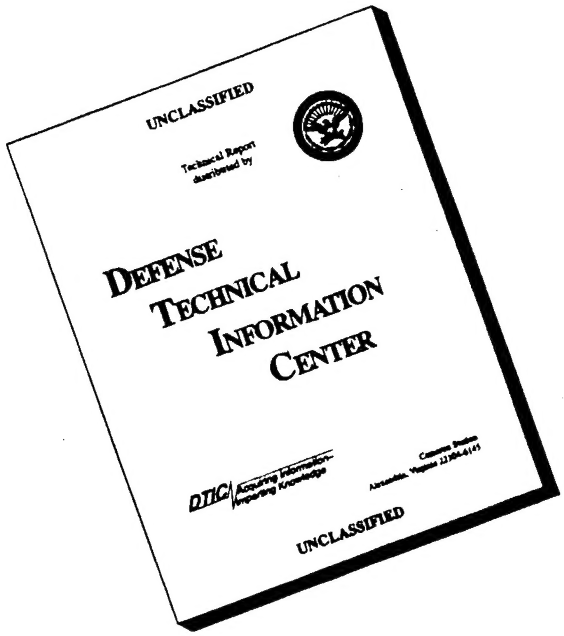

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DESIGN OF SHEAR DEFORMABLE ANTISYMMETRIC ANGLE-PLY LAMINATES TO MAXIMIZE THE FUNDAMENTAL FREQUENCY AND FREQUENCY SEPARATION

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ABSTRACT

An antisymmetrically laminated angle-ply plate is optimized with the objectives of maximizing the fundamental eigenfrequency and the distance between two consecutive natural frequencies. The formulation includes the contribution of the shear deformation, but neglects the in-plane and rotary inertias. The design variables are the fiber orientations and the thicknesses of individual layers. The design problems are cast into a mathematical programming format and solved by using a quasi-Newton function maximization algorithm. A penalty function method is employed to maximize the fundamental frequency, subject to lower bound constraints on higher order frequencies. Numerical results are presented for laminates constructed of high modulus fibre reinforced materials, and the effects of various problem parameters on the efficiency of the designs are investigated. It is shown that the design variables may not be determined optimally if the effect of shear deformation is neglected. Moreover, it was observed that the classical plate theory leads to erroneous results in optimal material selection problems.

1. INTRODUCTION

In a recent article Bert and Chen [1] studied the effect of shear deformation on the natural frequencies of an antisymmetric angle-ply laminate and showed that the classical plate theory (CPT) tends to overestimate the values of frequencies. Specifically, rectangular plates on simple supports were considered, and the effects of in-plane and rotary inertias were included in the formulation [1]. Subsequently, the same plate problem was also analyzed by means of a finite element method [2]. In the present article, a number of optimal design problems for similar structures are solved by employing the shear deformation theory (SDT) given in [1]. We neglect the effects of in-plane and rotary inertias, which were shown to have little effect on the fundamental frequency even in the case of relatively thick plates [1]. In particular, we consider the problems of maximizing the fundamental frequencies of angle-ply laminates with and without lower bounds on higher order frequencies and of maximizing the distance between two consecutive frequencies.

The design variables are taken as the fiber orientations and thicknesses of individual layers. Optimization problems are formulated as mathematical programming problems and solved by using a quasi-Newton nonlinear function maximization algorithm. In the constrained optimization problem, the lower bounds on higher order frequencies are incorporated into the formulation by means of a penalty function technique.

Optimal designs of laminated plates were given in several studies [3-8] with respect to natural frequencies in which the effect of shear deformation was neglected. We refer the reader to [7] for works concerning the optimization of laminates with respect to other objectives, such as

buckling loads and deflections. In the case of isotropic structures, shear deformation was taken into account in the optimal thickness design of a number of structural elements [9-14]. In these studies, it is observed that the shear deformation always reduces the efficiency of a design, the extent of which depends on the specific objective and the structure. Our results also confirm this general result. In fact, in the case of high modulus fiber reinforced materials used in high technology applications, the decrease in efficiency is considerable. Moreover, the neglect of shear deformation may lead to designs that are only suboptimal. Indeed, the values of optimum fiber orientations and layer thicknesses depend on the side-to-thickness ratio, and consequently SDT and CPT yield different optimum points. CPT can even give qualitatively incorrect answers when choosing a particular material to produce the most efficient design. Indeed, CPT indicates that the efficiency depends on the ratio of transverse and longitudinal Young's moduli in a monotonous fashion. When shear deformation is taken into account, this relation is no longer monotonous and it has a maximum point. Thus, the problem of optimal material selection can only be handled by SDT.

Increasing the distance between two consecutive eigenfrequencies is a practical design consideration for vibrating structures. It provides large gaps between natural frequencies, and thereby the possibility of resonance due to external excitations is reduced. In spite of the practical advantages, optimal frequency separation problems received relatively little attention in literature. Bronowicki et al [15] gave the design of ring-stiffened cylindrical shells for maximizing the separation between the lowest two natural frequencies. Pappas [16] studied the same problem with a different optimization algorithm. Designs

for the maximum separation of frequencies were obtained for isotropic beams by Olhoff [17,18] by maximizing the higher order frequencies. In the case of composite structures, there does not seem to exist any study on this problem. The present article gives designs for maximum distance between the lowest and the next lowest two natural frequencies. In [15,17,18], it was observed that the higher order frequencies approach each other when the structure is designed with respect to frequency separation. The same phenomenon is also observed in the present case and seems to be a general characteristics of such designs. We offer an explanation for this in terms of the topology of the frequency surfaces in the design space.

2. PROBLEM FORMULATION

We consider a simply supported rectangular plate of length a, width b, thickness h and mass density ρ. The plate is composed of an even number of orthotropic layers of thickness H_k, the fibers of which are oriented alternately at angles θ_k and $-\theta_\ell$ and are placed antisymmetrically with respect to the middle surface. Plates with these characteristics are commonly known as antisymmetric angle-ply laminates [1,2,7,8]. The equations of motion, which include the effect of shear deformation, were given in [1] for freely vibrating angle-ply laminates. We adopt the same formulation in the present study but neglect the effects of in-plane and rotary inertias. The equations are presented in the Appendix. We obtain a non-dimensional form of these equations by introducing the following dimensionless quantities:

$$x = X/a, \; y = Y/b, \; u = U/a, \; v = V/b, \; w = W/a$$

$$a_{ij} = A_{ij}/E_T h,\ b_{ij} = B_{ij}/E_T h^2,\ d_{ij} = D_{ij}/E_T h^3$$
$$r = a/b,\ p = a/h,\ h_k = H_k/h, \tag{1}$$

where i and j are integers and k refers to the layer number. In equation (1), X and Y are Cartesian coordinates parallel to the respective plate edges; U, V, W are the displacement components in the directions of the XYZ system; A_{ij}, B_{ij} and D_{ij} are the laminate stiffnesses given by

$$(A_{ij},B_{ij},D_{ij}) = \int_{-h/2}^{h/2} (1,Z,Z^2)Q_{ij}^{(k)}\, dZ, \tag{2}$$

where $Q_{ij}^{(k)}$ denotes the plane stress reduced stiffness components of the k-th layer given in the Appendix. E_T is the transverse modulus of elasticity, but could be replaced by any reference modulus. Introducing (1) into (A1), we obtain the following non-dimensional system of coupled partial differential equations governing the free vibration of the shear deformable laminate:

$$a_{11}u_{xx} + a_{66}r^2u_{yy} + (a_{12}+a_{66})v_{xy} + b_{16}p^{-1}\,\Psi_{xx} + b_{26}r^2p^{-1}\,\Psi_{yy} + 2b_{16}rp^{-1}\,\Phi_{xy} = 0 \tag{3}$$

$$(a_{12}+a_{66})u_{xy} + a_{66}r^{-2}v_{xx} + a_{22}v_{yy} + 2b_{26}p^{-1}\,\Psi_{xy} + b_{16}r^{-1}p^{-1}\Phi_{xx} + b_{26}rp^{-1}\,\Phi_{yy} = 0 \tag{4}$$

$$k_5^2\, a_{55}w_{xx} + k_4^2\, a_{44}r^{-2}w_{yy} + k_4^2\, a_{44}r\,\Psi_y + k_5^2\, a_{55}\,\Phi_x = \rho\, a^2E_T^{-1}\, w_{tt} \tag{5}$$

$$b_{16}p^{-1}u_{xx} + b_{26}r^2p^{-1}u_{yy} + 2b_{26}p^{-1}v_{xy} - k_4^2a_{44}rw_y$$
$$+ d_{66}p^{-2}\Psi_{xx} + d_{22}r^2p^{-2}\Psi_{yy} - k_4^2a_{44}\Psi \tag{6}$$
$$+ (d_{12}+d_{66})rp^{-2}\Phi_{xy} = 0$$

$$2b_{16}rp^{-1}u_{xy} + b_{16}r^{-1}p^{-1}v_{xx} + b_{26}rp^{-1}v_{yy}$$
$$-k_5^2a_{55}w_x + (d_{12}+d_{66})rp^{-2}\Psi_{xy} + d_{11}p^{-2}\Phi_{xx} \tag{7}$$
$$+ d_{66}r^2p^{-2}\Phi_{yy} - k_5^2a_{55}\Phi = 0$$

where Φ and Ψ are the slopes in xz and yz planes respectively; k_4 and k_5 are shear correction coefficients and t denotes time.

We consider the same simply supported plate boundary conditions as the ones given in [1]. In terms of dimensionless quantities, these are

$$u(0,y) = u(1,y) = 0,\ N_6(x,0) = N_6(x,1) = 0$$
$$N_6(0,y) = N_6(1,y) = 0,\ v(x,0) = v(x,1) = 0$$
$$w(0,y) = w(1,y) = w(x,0) = w(x,1) = 0$$
$$M_1(0,y) = M_1(1,y) = M_2(x,0) = M_2(x,1) = 0$$
$$\Psi(0,y) = \Psi(1,y) = 0,\ \Phi(x,0) = \Phi(x,1) = 0$$

where the dimensionless stress couples M_1 and M_2 and in-plane stress resultant N_6 are given by

$$N_6 = a_{66}(r^{-1}v_x+ru_y) + b_{16}p^{-1}\Phi_x + b_{26}rp^{-1}\Psi_y$$
$$M_1 = b_{16}(r^{-1}v_x+ru_y) + d_{11}p^{-1}\Phi_x + d_{12}rp^{-1}\Psi_y \tag{9}$$
$$M_2 = b_{26}(r^{-1}v_x+ru_y) + d_{12}p^{-1}\Phi_x + d_{22}rp^{-1}\Psi_y$$

The objectives of the study are to maximize the fundamental eigenfrequency Ω_1 and the distance between the consecutive natural frequencies of the

freely vibrating laminate by optimally determining the fiber orientation θ_k and the thickness h_k of the k-th layer. We note that the fundamental frequency Ω_{11} was found to be equal to the frequency of the eigenmode m=1, n = 1 for all problem parameters in the present problem. Here m and n denote the modal wave numbers associated with x and y directions. Consequently, Ω_1 will be denoted by Ω_{11} in the sequel. For higher order frequencies Ω_1, the values of m and n depend on the specific parameters. The designs for maximum fundamental frequency are classified as 'unconstrained' or 'constrained' depending on whether lower bounds on higher order frequencies exist. The design problem of maximizing the distance between consecutive frequencies is called the maximum frequency separation problem. Thus, we state the optimization problems as follows:

Unconstrained Design Problem : Determine the solution of the maximization problem

$$\max_{\theta_k, h_k} \Omega_{11} \ , \ k=1,2,\ldots,K/2, \tag{10}$$

where the design variables θ_k and h_k satisfy

$$0 \le \theta_k \le \pi/2 \text{ for } k = 1,3,\ldots,K-1, \ -\pi/2 \le \theta_k \le 0 \text{ for } k = 2,4,\ldots,K \tag{11}$$

$$\sum_{k=1}^{K} h_k = 1, \ h_k \ge 0, \ k = 1,2,\ldots,K, \tag{12}$$

with K denoting the total number of layers.

We note that due to antisymmetry, we have $\theta_k = -\theta_{K-k+1}$ and $h_k = h_{K-k+1}$. As a result of these requirements and (12), we have, in effect, K/2 variables of θ_k and (K/2)-1 variables of h_k. Thus, the total number of effective design variables is K-1.

Constrained Design Problem: Determine the solution of the maximization problem

$$\max_{\theta_k, h_k} \Omega_{11}\ , \ k=1,2,\ldots,K/2 \tag{13}$$

subject to

$$\Omega_{1_i} \geq \bar{\Omega}_{1_i}\ ,\ 1_i \geq 2,\ i=1,2,\ldots, \tag{14}$$

where θ_k and h_k satisfy (11) and (12) respectively and Ω_{1_i} denotes the 1_i-th order frequency. $\bar{\Omega}_{1_i}$ is a specified lower bound.

Design for maximum frequency separation: Determine the solution of maximization problem

$$\max_{\theta_k, h_k}(\Omega_1-\Omega_{1-1})\ ,\ k=1,2,\ldots,K/2, \tag{15}$$

where θ_k and h_k satisfy (11) and (12), respectively, and Ω_1 denotes the 1-th order frequency.

We observe that all designs problems are in fact nonlinear programming problems due to the nonlinear dependence of the eigenfrequencies on θ_k and h_k.

3. METHOD OF SOLUTION

The following set of displacement and rotation functions satisfies the differential equations (3)-(7) and the boundary conditions (8) [1]:

$$u = u_{mn} \sin m\pi x \cos n\pi y \cos \Omega_{mn} t$$
$$v = v_{mn} \cos m\pi x \sin n\pi y \cos \Omega_{mn} t$$
$$w = w_{mn} \sin m\pi x \sin n\pi y \cos \Omega_{mn} t$$
$$\Psi = \Psi_{mn} \sin m\pi x \cos n\pi y \cos \Omega_{mn} t$$
$$\Phi = \Phi_{mn} \cos m\pi x \sin n\pi y \cos \Omega_{mn} t$$

where Ω_{mn} is the frequency of the eigenmode (m,n).

Insertion of (16) into (3)-(7) leads to a set of linear homogeneous algebraic equations of the form:

$$C\underset{\sim}{S} = 0 \tag{17}$$

where $\underset{\sim}{S} = [u_{mn}\ v_{mn}\ w_{mn}\ \Psi_{mn}\ \phi_{mn}]^T$ and C is the 5x5 symmetric matrix, the elements of which are given by

$$
\begin{aligned}
c_{11} &= -a_{11}\alpha^2 - a_{66}r^2\beta^2, \quad c_{12} = -(a_{12}+a_{66})\alpha\beta \\
c_{13} &= 0, \quad c_{14} = -b_{16}p^{-1}\alpha^2 - b_{26}r^2p^{-1}\beta^2, \quad c_{15} = -2b_{16}rp^{-1}\alpha\beta \\
c_{22} &= -a_{66}r^{-2}\alpha^2 - a_{22}\beta^2, \quad c_{23} = 0, \quad c_{24} = -2b_{26}p^{-1}\alpha\beta \\
c_{25} &= -b_{16}r^{-1}p^{-1}\alpha^2 - b_{26}rp^{-1}\beta^2, \\
c_{33} &= -k_5^2a_{55}\alpha^2 - k_4^2a_{44}r^2\beta^2 + p^{-2}\omega_{mn}^2 \\
c_{34} &= -k_4^2a_{44}r\beta, \quad c_{35} = -k_5^2a_{55}\alpha \\
c_{44} &= -d_{66}p^{-2}\alpha^2 - d_{22}r^2p^{-2}\beta^2 - k_4^2a_{44} \\
c_{45} &= -(d_{12}+d_{66})rp^{-2}\alpha\beta \\
c_{55} &= -d_{11}p^{-2}\alpha^2 - d_{66}r^2p^{-2}\beta^2 - k_5^2a_{55}
\end{aligned}
$$

where $\alpha = m\pi$, $\beta = n\pi$ and the dimensionless eigenfrequency ω_{mn} relates to Ω_{mn} by the formula

$$\omega^2_{mn} = \rho\, a^4\Omega^2_{mn}/E_T h^2 \tag{19}$$

A nontrivial solution of (17) exists if

$$\det C = 0, \tag{20}$$

which leads to an explicit expression for ω_{mn}, viz.

$$\omega_{mn}^2 = p^2F_{33}^{-1} \left(k_5^2 a_{55}\alpha(F_{35}+\alpha F_{33}) + k_4^2 a_{44} r\beta(F_{34}+r\beta F_{33})\right) \quad (21)$$

where F_{ij} denotes the cofactor of the element C_{ij}. We note that an explicit expression for ω_{mn} can be obtained due to the fact that the in-plane and rotary inertias are neglected. Otherwise the determinental equation (20) becomes a fifth order polynomial in ω_{mn}^2 [1].

Maximization of the expression (21) with m=n=1 and with respect to the design variables θ_k and h_k, which are subject to the constraints (11) and (12), leads to the solution of the unconstrained design problem (10). In the case of the constrained design, the fundamental frequency ω_{11} is to be maximized subject to constraints $\omega_{\ell_i} \geq \bar{\omega}_{\ell_i}$, $\ell_i > 2$, i=1,2,... on ℓ_i-th order frequencies ω_{ℓ_i}. This type of problem was already treated by Adali [8,19] in the context of laminated plates and vibrating beams by employing a penalty function method. The procedure converts the constrained optimization problem into an unconstrained one by treating the programming problem

$$\max_{\theta_k,\, h_k} \{\omega_{11} + \sum_{i=1}^{I} \frac{1}{\epsilon_i} \max [0, \bar{\omega}_{\ell_i} - \omega_{\ell_i}]\} , \quad (22)$$

where $\epsilon_i > 0$ are specified small constants. For further details on the penalty function method we refer the reader to [19]. Solution of problem (22), subject to (11) and (12) provides the constrained optimal design of the laminate.

We note that both the constrained design problem and the maximum frequency separation problem involve the evaluation of higher order frequencies. Although the fundamental frequency is given by ω_{11} for all the problem parameters, the modal wave numbers m and n depend on the specific problem parameters in the case of higher order frequencies. We compute the ℓ-th order frequency ω_ℓ from

$$\omega_\ell = \min_{m,n} \omega_{mn} \text{ subject to } \omega_{mn} \geq \omega_{\ell-1} \quad . \quad (23)$$

which constitutes a discrete minimization problem over the integers m and n.

4. DESIGN FOR MAXIMUM FUNDAMENTAL FREQUENCY

In this section, numerical results for the optimal design of an angle-ply laminate for maximum ω_{11} are given, and the effect of various problem parameters on the efficiency of a design is investigated. We treat two different kinds of graphite epoxy plastic with material constants given by

Material I: $E_L/E_T = 40.0, \quad G_{LT}/E_T = 0.6$

$G_{TZ}/E_T = 0.5, \quad \nu_{LT} = 0.25$

and

Material II: $E_L/E_T = 25.0, \quad G_{LT}/E_T = 0.5$

$G_{TZ}/E_T = 0.2, \quad \nu_{LT} = 0.25.$

The shear coefficients are taken as $k_4^2 = k_5^2 = 5/6$.

The efficiency of an optimal design is assessed by comparing it with the corresponding standard plate, which we define to be composed of layers of equal thickness with the fibers oriented alternately at 45 and -45 degrees. The efficiency index is defined as

$$E_{ff} = 100((\omega_{op}/\omega_s) - 1), \tag{24}$$

which gives the percent increase in the fundamental frequency ω_{op} of the optimal plate as compared to the fundamental frequency ω_s of the standard plate.

Fig.1 gives the efficiency curves plotted against the aspect ratio a/b for four-layered laminates of material I, optimized with respect to fiber orientations only. The results at a/b = 0 refer to a plate strip, and

those for $a/h = \infty$ are obtained by employing the classical plate theory. We observe that the effect of shear deformation is to decrease the efficiency of the design and this effect becomes more pronounced as the a/h ratio becomes smaller. Exactly the same situation was observed in the optimal thickness design of one-dimensional structures that included shear deformation [9-12].

Fig.2 gives the efficiency curves for laminates made of material II with the same problem parameters as in Fig.1. A comparison of Figs.1 and 2 indicates that material I produces less efficient designs for $a/h < 10$ when $a/b < 1$ and for $a/h < 40$ when $a/b > 1$ than material II, but this is not the case for $a/h = \infty$. An important implication of this observation is that an assessment of efficiency of plates constructed from different materials would be completely misleading if the shear deformation were neglected.

Tables 1 and 2 provide the values of fundamental frequencies and fiber orientations of optimally designed laminates made of materials I and II respectively. We note that the results for optimum θ_i's are given for only one-half of the laminates because of the antisymmetry. Table 1 indicates that the fiber orientations of optimal plates depend on the a/h ratio, and consequently SDT and CPT yield different results. We observe from Table 2 that these differences are less pronounced for material II. Thus the extent of the difference in optimum θ_i's obtained by SDP and CPT is closely related to the material properties and mostly to E_L/E_T ratio.

Next, we investigate the amount of decrease in the maximum fundamental frequencies of optimal designs as a result of taking shear deformation into account. Let ω_{SD} and ω_{CP} denote the fundamental frequencies of optimal laminates obtained by SDT and CPT respectively. Then, the percent decrease in efficiency as a result of employing SDT rather than CPT is

given by

$$D_{eff} = 100(1 - \omega_{SD} / \omega_{CP}). \tag{25}$$

The curves of D_{eff} plotted against the aspect ratio are given in Fig.3 for four-layered laminates optimized with respect to θ_i. We observe that the efficiencies obtained from CPT become less accurate as a/h decreases and/or a/b increases. Moreover, the material properties have relatively small effect on the final result.

In the above results, the laminates were optimized with respect to fiber orientations only. The effect of including the layer thicknesses among the design variables is studied in Fig.4, which shows the differences in the efficiencies of optimal plates with design variables (θ_i, h_i) and (θ_i). In the particular case of four-layered plates considered in Fig.4, the specific design variables are $(\theta_1, \theta_2, h_1)$ and (θ_1, θ_2). The percent diffe= rences in the efficiencies are given by

$$100(\omega_{op}^{(3)} - \omega_{op}^{(2)})/\omega_s,$$

where $\omega_{op}^{(3)}$ and $\omega_{op}^{(2)}$ denote fundamental frequencies of optimal laminates with three and two design variables respectively. We observe that the optimum layer thickness becomes most effective when aspect ratios are close to unity, and moreover the increase in the efficiency depends heavily on a/h ratio. Another interesting conclusion is that the optimum deter= mination of the layer thickness increases the efficiency of a laminate for aspect ratios where CPT would indicate no change. More specifically CPT shows no increase in the efficiency of a design as a result of including the layer thickness among the design variables when $0.55 > a/b > 1.7$.

Table 3 gives the values of the maximum ω_{11} and optimum $\theta_1, \theta_2, h_2 (=z(3)=-z(1))$ for various a/h and a/b ratios and for laminates made of material I. A comparison of Tables 1 and 3 indicates that optimum values of θ_i depend on

the number of design variables.

Fig.5 gives the curves of efficiency plotted against a/h for laminates with two, four and six layers. We observe that the efficiency drops as the number of layers increases. Moreover, the efficiencies of Materials I and II may be greater or less than one another depending on a/h as illustrated in the case of four-layered laminates. Thus we again come to the conclusion that a comparison of the efficiencies of different materials by CPT alone is likely to give incorrect results for thick laminates.

The effect of the modulus ratio E_L/E_T on the efficiency is specifically investigated in Fig.6, where the efficiency curves are plotted against E_L/E_T for laminates optimized with respect to θ_i and with $G_{LT}/E_T = 0.6$, $G_{TZ}/E_T = 0.5$, $\nu_{LT} = 0.25$, r=2 and number of layers 4. We observe that the efficiency decreases after a certain value of E_L/E_T as E_L/E_T increases when $a/h < 40$. On the other hand CPT indicates a steady increase in efficiency for the range of values of E_L/E_T given in Fig.6. Thus, even a qualitative estimate based on CPT as to the efficiency of laminates made of different materials will be incorrect. From Fig.6, it appears that the effect of shear deformation increases as E_L/E_T increases, causing the efficiency to drop after a certain point. The initial increase in efficiency apparently occurs when the deformation due to bending only dominates the contribution of the shear deformation.

5. CONSTRAINED OPTIMIZATION AND OPTIMAL FREQUENCY SEPARATION

In many applications, the maximization of the fundamental frequency may be subjected to additional design requirements in the form of constraints on higher order frequencies. Usually one or more of the higher order frequencies are required to be greater than certain values. Another class

of problems is the design of a laminate that has maximum separation between the consecutive frequencies. The optimization technique used in the present study can be easily extended to solve both problems.

Table 4 gives the values of the maximum fundamental frequencies with second and third order frequencies subjected to lower bounds $\bar{\omega}_2$ and $\bar{\omega}_3$ respectively. The results are given for four-layered laminates optimized with respect to θ_i only, with a/h = 10.0 and a/b = 1,2. When $\bar{\omega}_2$ = 24.0 and $\bar{\omega}_3 = \infty$, with a/b = 1, the laminate has a double eigenvalue as its second order frequency. This situation changes when we set $\bar{\omega}_3$ = 37.0, and moreover the mode of the third order frequency changes from (2,1) to (1,3) as $\bar{\omega}_3$ is increased to $\bar{\omega}_3$ = 38.0. The results for r=2 indicate that ω_2 and ω_3 can be increased considerably with little effect on ω_{11}.

Numerical results for the optimal frequency separation problem are given in Table 5 for similar parameters. We observe that the maximizations of $\omega_2-\omega_{11}$ with r=1 and $\omega_3-\omega_2$ with r=1,2 lead to double eigenvalues for the second and third order frequencies respectively. This may be attributed to the fact that a frequency surface in the design space usually has a local maximum at those points where the surfaces of different eigenmodes cross each other. Obviously these points are also those where double eigenvalues occur.

6. CONCLUSIONS

The main effect of the shear deformation on an optimally designed laminate is to reduce its efficiency. We note that the efficiency is defined in comparison to the corresponding antisymmetric angle-ply laminate that has equal thickness layers with fibers oriented at alternating angles of 45 degrees. The efficiency decreases with decreasing side-to-thickness ratio, as is the case in the optimal thickness design of isotropic structures [9-12]. The aspect ratio, number of layers and the material properties have considerable

effect on the efficiency of a design, as depicted in Figs. 1,2,5 and 6.

Comparison of results obtained by SDT and CPT yields interesting insights into the ways the shear deformation affects the optimal laminates made of advanced filamentary composite materials. For example, SDT and CPT produce different values for optimum fiber orientations and layer thicknesses. Thus, a design based on CPT alone could, in fact, be only suboptimal. Deviation from the optimum increases with decreasing side-to-thickness and increasing E_L/E_T ratios when CPT is employed.

Furthermore, CPT gives qualitatively incorrect results when a comparison of different composite materials is made. Indeed, CPT indicates that the efficiency would increase with increasing E_L/E_T ratio. Therefore, the material with the highest E_L/E_T yields the most efficient design according to CPT. In fact, the efficiency decreases after a certain point with increasing E_L/E_T when the shear is taken into account. Thus, SDT indicates an optimal E_L/E_T value for the maximum efficiency.

The efficiency can be increased by including the layer thicknesses among the fiber orientations as additional design variables. This increase is larger when the side-to-thickness ratio is high and when the aspect ratio is close to unity, but it becomes negligible if the aspect ratio is too small or too large. CPT yields misleading results on this aspect by indicating too narrow a range for the aspect ratio around unity where optimizing with respect to layer thicknesses can be effective.

Constrained designs of the laminates were obtained when lower bounds were imposed on higher order natural frequencies. It was observed that the imposed bounds effected the eigenmode of the higher order frequencies at the optimum point (Table 4).

Designs for optimal frequency separation lead to double eigenvalues for

higher order frequencies in many cases (Table 5). This seems to be due to local maxima, which occur at the cross sections of surfaces formed by different eigenmodes in the design space.

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/MR

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APPENDIX

The equations of motion governing the free vibrations of an antisymmetric angle-ply laminated plate are given by [1]:

$$L\,D = 0, \qquad \text{(A1)}$$

where $D = [U\ V\ W\ h\Psi\ h\Phi]^T$ is the transpose of the displacement vector and $L = [L_{k\ell}]$ is the 5 by 5 symmetric differential operator matrix, the components of which are given by

$$L_{11} = A_{11}\,D_X^2 + A_{66}\,D_Y^2, \quad L_{12} = (A_{12} + A_{66})D_X\,D_Y$$
$$L_{13} = 0, \; L_{14} = (B_{16}/h)D_X^2 + (B_{26}/h)D_Y^2$$
$$L_{15} = (2B_{16}/h)D_XD_Y, \quad L_{22} = A_{66}D_X^2 + A_{22}\,D_Y^2$$
$$L_{23} = 0, \quad L_{24} = (2B_{26}/h)D_XD_Y, \quad L_{25} = L_{14}$$
$$L_{33} = -k_5^2\,A_{55}\,D_X^2 - k_4^2\,A_{44}\,D_Y^2 + \rho\,h\,D_t^2$$
$$L_{34} = -(k_4^2\,A_{44}/h)D_Y, \; L_{35} = -(k_5^2\,A_{55}/h)D_X^2$$
$$L_{44} = (D_{66}/h^2)D_X^2 + (D_{22}/h^2)D_Y^2 - k_4^2\,A_{44}/h^2$$
$$L_{45} = ((D_{12} + D_{66})/h^2)D_X\,D_Y$$
$$L_{55} = (D_{11}/h^2)D_X^2 + (D_{66}/h^2)D_Y^2 - k_5^2\,A_{55}/h^2,$$

where $D_X \equiv \partial/\partial X$, $D_Y \equiv \partial/\partial Y$, $D_t = \partial/\partial t$.

Plane stress reduced stiffness components of the k-th layer are given by

$$Q_{11}^{(k)} = Q_{11}\,c^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}\,s^4$$
$$Q_{12}^{(k)} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(s^4 + c^4)$$
$$Q_{22}^{(k)} = Q_{11}\,s^4 + 2(Q_{12} + 2Q_{66})s^2c^2 + Q_{22}\,c^4 \qquad \text{(A2)}$$
$$Q_{16}^{(k)} = (Q_{11} - Q_{12} - 2Q_{66})s\,c^3 + (Q_{12} - Q_{22} + 2Q_{66})s^3c$$
$$Q_{26}^{(k)} = (Q_{11} - Q_{12} - 2Q_{66})s^3c + (Q_{12} - Q_{22} + 2Q_{66})s\,c^3$$

$$Q_{44}^{(k)} = Q_{44}\, c^2 + Q_{55}\, s^2$$
$$Q_{55}^{(k)} = Q_{44}\, s^2 + Q_{55}\, c^2$$
$$Q_{66}^{(k)} = (Q_{11}+Q_{22}-2Q_{12}-2Q_{66})s^2c^2+Q_{66}(s^4+c^4),$$

where $c = \cos \theta_k$, $s = \sin \theta_k$,

$$Q_{11} = E_L/(1-\nu_{LT}\nu_{TL}),\ Q_{12}= \nu_{LT}E_T/(1-\nu_{LT}\nu_{TL})$$
$$Q_{22} = E_T/(1-\nu_{LT}\nu_{TL}),\ Q_{44} = G_{TZ}$$
$$Q_{55} = Q_{66} = G_{LT},\ \ \nu_{TL} = \nu_{LT}\, E_T/E_L.$$

Here E_L and E_T denote Young's moduli in the longitudinal and transverse directions, respectively; ν_{LT} is the ratio of transverse-to-longitudinal strain under longitudinal stress; G_{LT} and G_{TZ} denote in-plane and thickness shear moduli respectively.

TABLE 1: Maximum fundamental eigenfrequencies and optimum fiber orientations in degrees of a four-layered angle-ply plate made of material I, with $h_k = 0.25$, $k=1,2,3,4$.

$\frac{a}{h}$ / $\frac{a}{b}$	5	10	20	40	∞
0.0	9.46 (0.0)*	14.00 (0.0)	16.71 (0.0)	17.67 (0.0)	18.03 (0.0)
0.5	9.73 (14.4/ -30.0)	14.16 (5.2/ -15.8)	16.88 (0.0)	17.85 (0.0)	18.21 (0.0)
1.0	12.67 (45.0/ -45.0)	18.53 (45.0/ -45.0)	21.91 (45.0/ -45.0)	23.09 (45.0/ -45.0)	23.53 (45.0/ -45.0)
2.0	22.55 (68.6/ -61.4)	38.93 (75.6/ -60.0)	56.62 (84.8/ -74.2)	67.50 (90.0)	72.84 (90.0)
4.0	44.57 (81.0/ -77.2)	85.46 (85.2/ -78.6)	151.79 (90.0)	224.56 (90.0)	289.70 (90.0)
8.0	89.04 (90.0)	176.19 (90.0)	340.18 (90.0)	509.10 (90.0)	1154.80 (90.0)

* A single angle applies to all layers.

TABLE 2: Maximum fundamental eigenfrequency and optimum fiber orientations in degrees of a four-layered angle-ply plate made of material II, with $h_k = 0.25$, k=1,2,3,4.

$\frac{a}{h}$ / $\frac{a}{b}$	5	10	20	40	∞
0.0	8.26 (0.0)*	11.67 (0.0)	13.46 (0.0)	14.05 (0.0)	14.26 (0.0)
0.5	8.40 (0.0)	11.83 (0.0)	13.64 (0.0)	14.24 (0.0)	14.46 (0.0)
1.0	10.12 (45.0/ -45.0)	14.83 (45.0/ -45.0)	17.55 (45.0/ -45.0)	18.50 (45.0/ -45.0)	18.86 (45.0/ -45.0)
2.0	19.59 (87.8/ -86.0)	33.61 (90.0)	47.32 (90.0)	54.56 (90.0)	57.83 (90.0)
4.0	40.29 (90.0)	76.86 (90.0)	132.72 (90.0)	187.27 (90.0)	223.94 (90.0)
8.0	81.02 (90.0)	160.08 (90.0)	306.14 (90.0)	529.38 (90.0)	913.57 (90.0)

* A single angle applies to all layers.

TABLE 3: Maximum fundamental eigenfrequencies, optimum fiber orientations and z(3) = -z(1) coordinates of a four layered angle-ply plate made of material I

$\frac{a}{h}$ / $\frac{a}{b}$	5	10	20	40	∞
0.0	9.88 (25.9/ -17.7, 0.38)	14.19 (16.0/ -10.2, 0.39)	**	**	**
1.0	13.00 (45.0/ -45.0, 0.35)	19.59 (45.0/ -45.0, 0.35)	23.73 (45.0/ -45.0, 0.35)	25.25 (45.0/ -45.0, 0.35)	25.82 (45.0/ -45.0, 0.35)
2.0	22.80 (59.4/ -73.3, 0.39)	39.52 (64.1/ -72.3, 0.38)	56.77 (74.0/ -79.8, 0.39)	**	**
4.0	44.72 (64.3/ -85.2, 0.45)	85.77 (67.6/ -84.7, 0.44)	**	**	**

** The values are the same as those given in Table I and the z(3) = - z(1) coordinate has no effect on the value of ω .

TABLE 4: The values of the fundamental (ω_{11}), second (ω_2) and third (ω_3) order frequencies of an optimal four-layered angle-ply plate of material I subject to constraints on the second ($\bar{\omega}_2$) and/or third ($\bar{\omega}_3$) order frequencies, with a/h = 10.0

Constraints		Eigenfrequencies			Optimum fiber orientations	
$\bar{\omega}_2$	$\bar{\omega}_3$	ω_{11}	ω_2	ω_3	θ_1	θ_2
			r = 1			
24.0	–	18.53	35.06 (1,2)*	35.06 (2,1)	45.0	-45.0
24.0	37.0	15.24	32.29 (1,2)	37.00 (2,1)	36.7	-35.8
24.0	38.0	16.43	24.71 (1,2)	38.00 (1,3)	15.1	-32.7
			r = 2			
44.0	–	38.79	44.00 (2,1)	52.69 (3,1)	80.3	-56.5
44.0	58.0	38.64	46.62 (2,1)	58.00 (3,1)	69.3	-50.0

* The numbers in parentheses denote (m,n).

TABLE 5: Maximum separations between first and second ($\omega_2 - \omega_{11}$) and second and third ($\omega_3 - \omega_2$) order eigenfrequencies of a four-layered angle-ply of material I, with a/h = 10.0.

Objective	ω_{11}	ω_2	ω_3	$\omega_2 - \omega_{11}$	$\omega_3 - \omega_2$
			r = 1		
$\omega_2 - \omega_{11}$	18.53	35.06 (1,2)*	35.06 (2,1)	16.53	0.0
	(θ_1 = 45.0, θ_2 = - 45.0)				
$\omega_3 - \omega_2$	14.74	22.86 (1,2)	37.35 (1,3) (2,1)	8.12	14.48
	(θ_1 = 0.0, θ_2 = - 55.8)				
			r = 2		
$\omega_2 - \omega_{11}$	19.21	40.90 (2,1)	43.94 (1,2)	21.70	3.04
	(θ_1 = θ_2 = 0.0)				
$\omega_3 - \omega_2$	29.97	45.98 (2,1)	67.04 (3,1) (1,2)	16.02	21.06
	(θ_1 = 23.9, θ_2 = - 55.4)				

* The numbers in parentheses denote (m,n).

LIST OF FIGURES

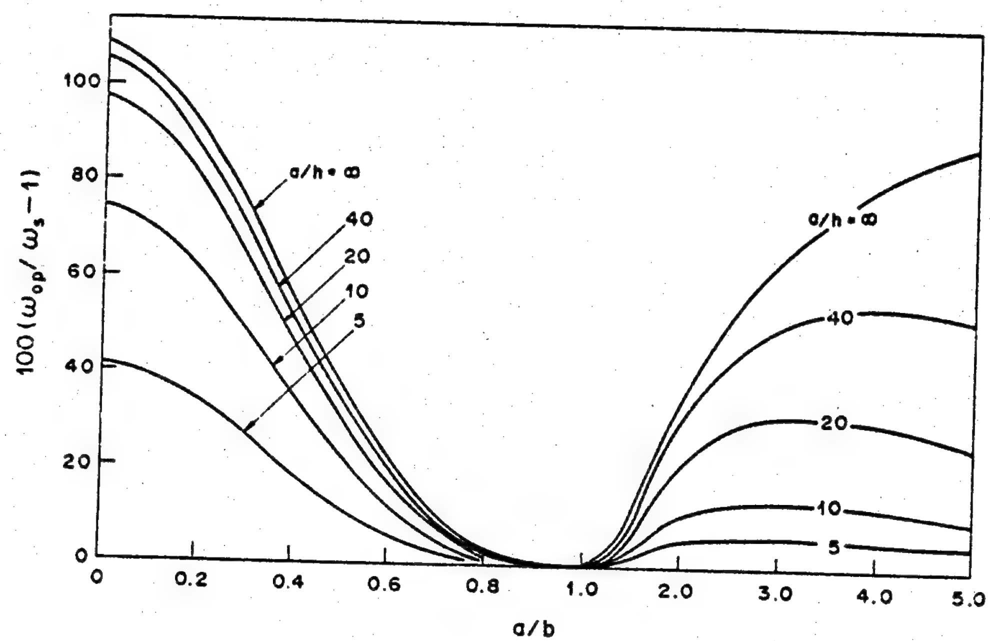

100(ω_op/ω_s−1)
a/b
a/h=∞
40
20
10
5
0
0.2
0.4
0.6
0.8
1.0
2.0
3.0
4.0
5.0
100
80
60
40
20

FIG. 1

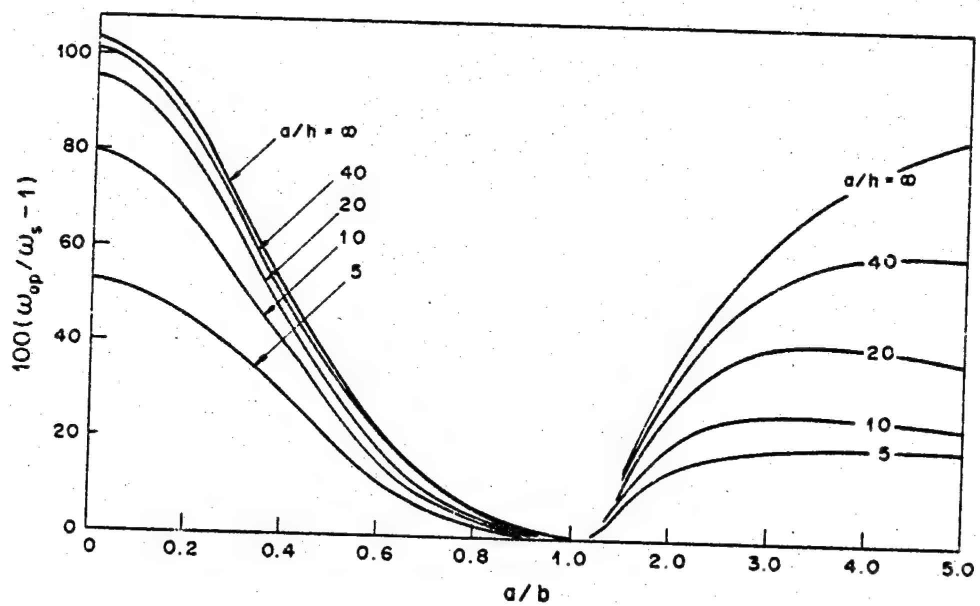
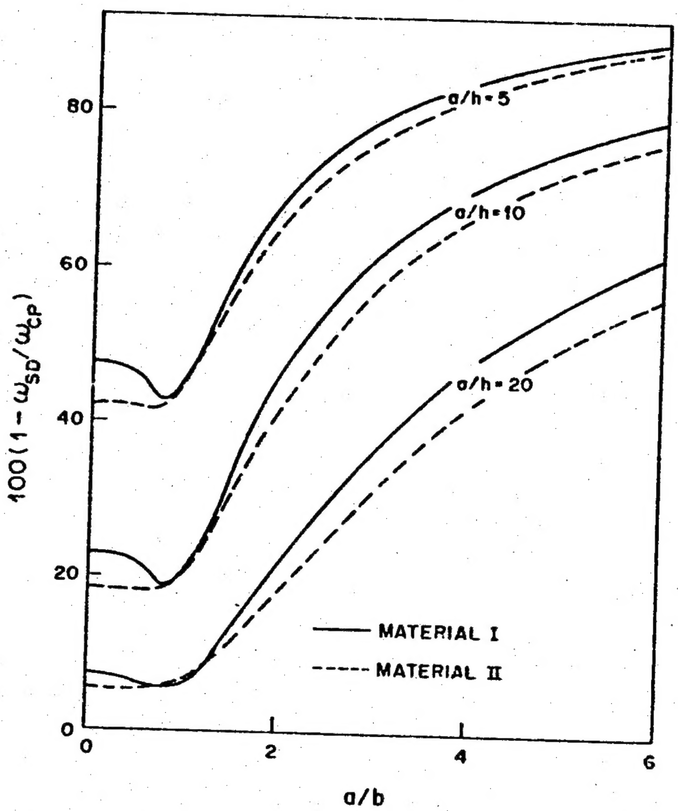

100(ω_op/ω_s - 1)
100
80
60
40
20
0
0
0.2
0.4
0.6
0.8
1.0
2.0
3.0
4.0
5.0
a/b
a/h = ∞
40
20
10
5
a/h = ∞
40
20
10
5

FIG.2

25



100(1 − ω_{SD}/ω_{CP})
80
60
40
20
0
0
2
4
6
a/b
a/h=5
a/h=10
a/h=20
MATERIAL I
MATERIAL II

FIG. 3

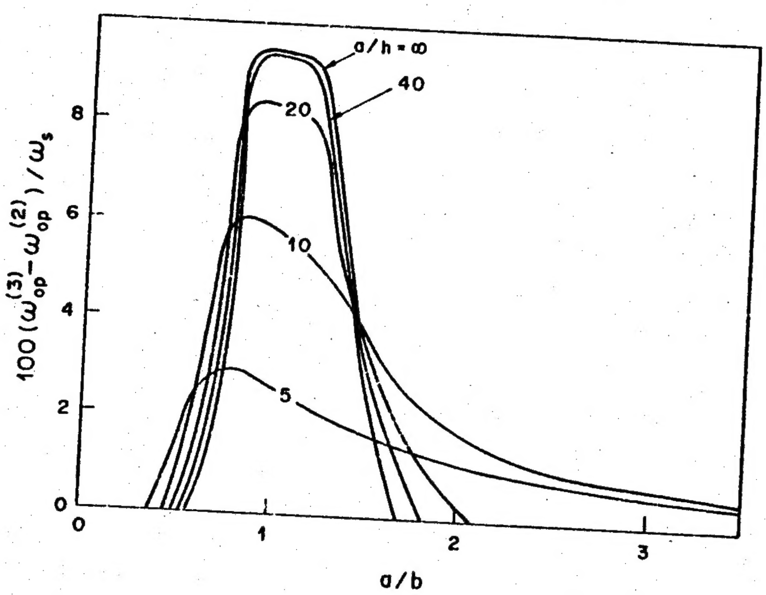
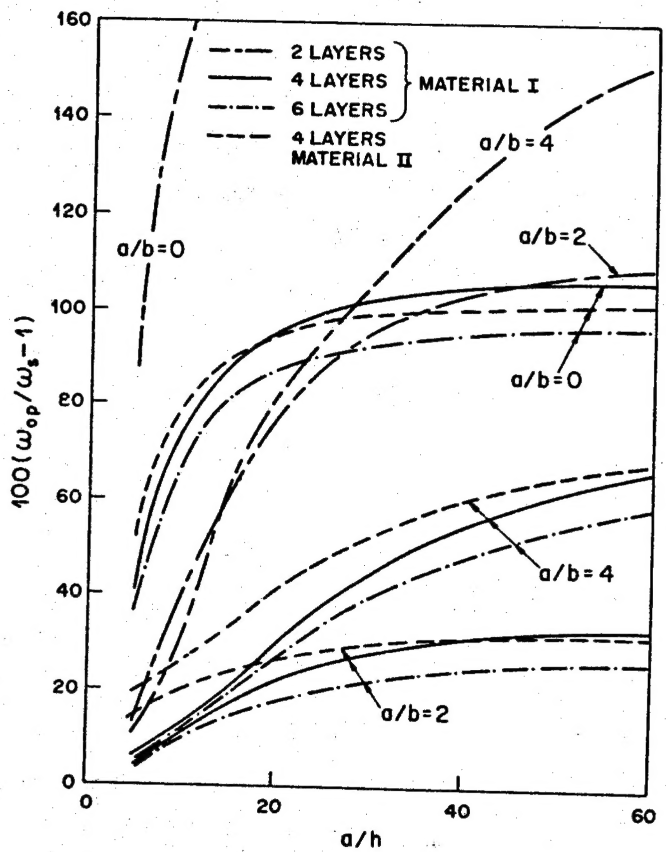


FIG. 4



2 LAYERS
4 LAYERS
6 LAYERS
MATERIAL I
4 LAYERS
MATERIAL II
a/b=0
a/b=4
a/b=2
a/b=0
a/b=4
a/b=2
160
140
120
100
80
60
40
20
0
0
20
40
60
$100(\omega_{op}/\omega_s-1)$
a/h

FIG.5

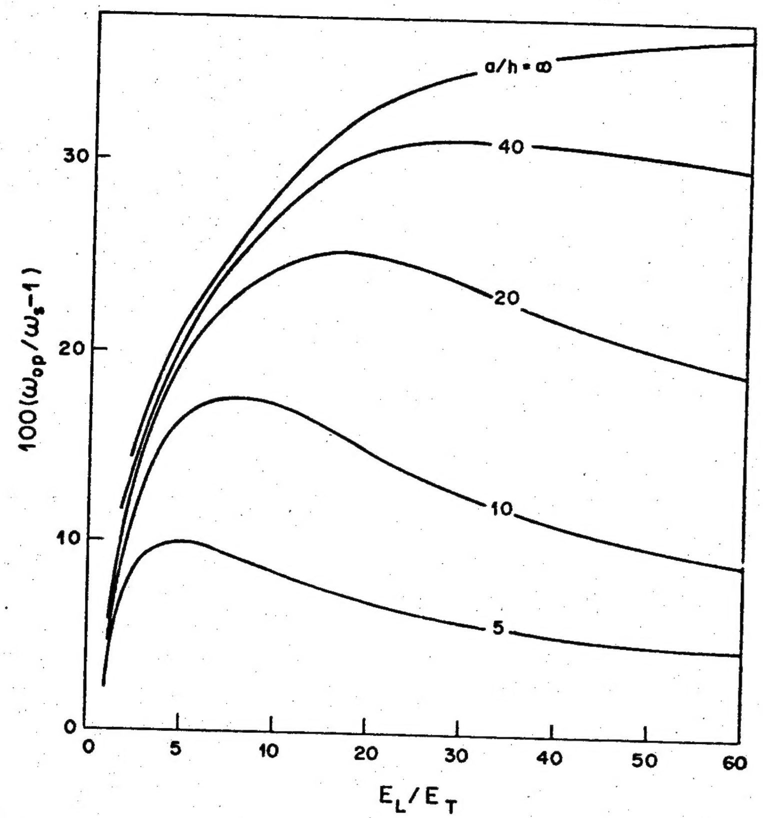


FIG. 6